Proposal title:

Resonant effects for Euler equations for rotating fluids

Project summary

Uniform rotation of incompressible fluids leads to generation of fast inertial waves which interact through the Euler nonlinearly. The interactions strongly depend on boundary conditions, we will study this dependence.

The research is to be concentrated on the following four types of problems:

1. Euler and Navier-Stokes equations with periodic boundary conditions (the case of resonant aspect ratios)
2. Rotating thin layers
3. Navier-Stokes equations with rapidly rotating initial fields in the unbounded domain
4. Euler equations for flows in a rotating bounded circular cylinder.

In 1, the problem is to find an extension of the results of Babin, Mahalov, Nicolaenko (henceforth BMN) [12] concerning integrability of the equations and regularity of their solutions to domains which have resonant aspect ratios, that is to the case when the limit equations for fast modes have non-trivial non-linear interactions.

In 2, the problem is to combine the techniques of BMN [12] with techniques of Raugel-Sell [47] and Avrin [4] in a situation when the rotating layer is thin to obtain optimal results on regularity.

In 3, the problem is to develope techniques to study and to prove global regularity of solutions of classical Navier-Stokes problem when initial data are a finite energy perturbation of a rapidly uniformly rotating fluid in the infinite space or in a horizontal layer.

In 4, the problem is to study which results of BMN [12] on integrability of Euler equations and global regularity of their solutions for uniformly rotating fluids with periodic boundary conditions can be extended to the case of the circular cylindrical boundary.

Among anticipated results of the research are:

(a) Derivation of the classical 3-wave equations as subsystems in a singular limit for rapidly rotating fluids.

(b) Partial solution for the classical problem of regularity of solutions of the Navier-Stokes system with large initial data. Namely, if a velocity field in unbounded space with a large constant curl is finitely perturbed at initial moment of time, then the velocity field remains smooth for all times.

(c) Derivation of limit equations for rapidly rotating fluids in rotation-invariant domains (up to now such equations were rigorously derived only for the periodic boundary conditions).

(d) Extension of the results of BMN on regularity to the case of non-periodic or resonant periodic boundary conditions.

In the proposed project we restrict ourselves to rotating fluids, stably stratified fluids may be studied similarly.