**Topic Class in Representation Theory (MATH 739)**

**Syllabus**

- Preliminary definitions: representations, equivalence, construction of new representations
- Invariant sub-spaces, irreducible representations, Shur's lemma
- Indecomposable representations, Mascke's theorem
- Complete reducibility of representations, decomposition of the regular representation; representations of the symmetric group $S_3$
- Characters (definition, properties, examples); conjugacy classes (examples: dihedral group, symmetric group, group of even permutations)
- Orthogonality of characters
- The number of irreducible characters
- The number of linear characters; character table of the symmetric group $S_4$
- Various problems on character tables
- Restriction of characters: theory and examples
- Induced representations
- Frobenius character formula
- Additional results on restricted and induced representations; Mackey's irreducibility criterion;
- Semi-direct product by an abelian normal subgroup
- Representations of the symmetric group: an introduction
- Specht modules; bases of Specht modules
- Ordering of partitions; classification of irreducible representations of $S_n$
- Branching rules; ring of symmetric functions, monomial and elementary symmetric functions, power sums
- Complete symmetric functions and Shur's functions; alternative definition of Shur's functions; Cauchy's formula; Frobenius character formula
- The ring of representation of $S_n$ and the ring of symmetric functions.