MATH 117, DYNAMICAL SYSTEMS HOMEWORK #1

Exercises 2.2, 2.6, and the following problems:

Problem 1.

Let $f : [0, 1] \to [0, 1]$ be a homeomorphism (i.e. f is a continuous bijection and f^{-1} is also continuous) such that $f(0) = 1$ and $f(1) = 0$. Prove that f has exactly one fixed point.

Problem 2.

Suppose that each of the rotations of the circle $R_\alpha : S^1 \to S^1$ and $R_\beta : S^1 \to S^1$ is minimal (i.e. every orbit is dense in the phase space). Does it imply that their product $f = R_\alpha \times R_\beta : \mathbb{T}^2 \to \mathbb{T}^2$ is also minimal? Explain your answer.

Problem 3.

Let $S \subseteq \Sigma_2^+$ be the set of all sequences of $0'$s and $1'$s hat do not contain more than 5 subsequent zeros.

a) Is the set S dense in Σ_2^+?

b) Give an example of a periodic point of the topological Bernoulli shift $\sigma : \Sigma_2^+ \to \Sigma_2^+$ that is contained in S, and of a periodic point that is not contained in S.

c) Prove the set S contains a dense set of periodic points of the topological Bernoulli shift.