MATH 117, DYNAMICAL SYSTEMS HOMEWORK #2

Exercises 2.17, 3.3, 3.9, and the following problems:

Problem 1.

Consider the map $E_3 : S^1 \to S^1$, $E_3(x) = 3x \pmod 1$. Set $x = \sum_{n=1}^{\infty} \frac{2}{3^{n^2}}$. Describe $\omega(x)$.

Problem 2.

Consider the map $E_2 : S^1 \to S^1$, $E_2(x) = 2x \pmod 1$. Prove that

a) there exists a point $x \in S^1$ such that $\omega(x)$ is a finite set that contains exactly 2020 points;

b) there exists a point $x \in S^1$ such that $\omega(x) = S^1$;

c) there exists a point $x \in S^1$ such that $\omega(x)$ is a Cantor set (i.e. non-empty closed perfect nowhere dense set);

d) there exists a point $x \in S^1$ such that $\omega(x)$ is countably infinite;

e) if $\omega(x)$ contains an interval, then $\omega(x) = S^1$.