MATH 117, DYNAMICAL SYSTEMS HOMEWORK #6

Exercises 5.12, 6.3, and these problems:

Problem 1.

Let $f : S^1 \to S^1$ be a continuous map of degree one that has a fixed point, $f(x_0) = x_0$ for some $x_0 \in S^1$. Assume also that f has a periodic point of prime period 3. Is it true that f must have periodic orbits of all periods? Prove or give a counterexample.

Problem 2.

Assume that all prime (i.e. smallest) periods of periodic orbits of a continuous map $f : [0, 1] \to [0, 1]$ are uniformly bounded (i.e. there exists $N \in \mathbb{N}$ such that the prime period of every periodic orbit of f is smaller than N). What can you say about periods of periodic orbits of f? For example, can f have a periodic orbit of period 2020? Of period 2048?

Problem 3.

Consider the following homeomorphism of the circle:

$$f(x) = \begin{cases} \frac{1}{4} + 2x \pmod 1, & \text{if } x \in [0, \frac{1}{4}]; \\ \frac{5}{8} + \frac{x}{2} \pmod 1, & \text{if } x \in [\frac{1}{4}, \frac{3}{4}]; \\ x + \frac{1}{4} \pmod 1, & \text{if } x \in [\frac{3}{4}, 1]. \end{cases}$$

What is the rotation number of f?