Problem 1.
Find all singular points of the vector field
\[
\begin{align*}
\dot{x} &= \sin y \\
\dot{y} &= \sin x
\end{align*}
\]
and determine their stability (both Lyapunov and asymptotic).

Problem 2.
Plot the phase portrait for the Newton equation
\[
\ddot{x} = -4x^3 + 4x
\]

Problem 3.
Let \( f : [0, 1] \rightarrow [0, 1] \) be a continuous mapping. Is it possible that \( f \) has a periodic orbit of prime period 2011, but does not have a periodic orbit of prime period 2010?

Problem 4.
Find all fixed points of the map \( f : \mathbb{R}^1 \rightarrow \mathbb{R}^1, f(x) = 2x - x^2 \), and determine their stability.

Problem 5.
Find the box counting dimension of the subset of \([0, 1]\) consisting of real numbers without digit "7" in their decimal expansion.
Problem 1.
Show that for every value of the real parameter $\mu$ the singular point $(0, 0)$ of the system
\[
\begin{align*}
\dot{x} &= 2x + (1 + \mu)y \\
\dot{y} &= (1 - \mu)x + y
\end{align*}
\]
is not Lyapunov stable.

Problem 2.
Plot the phase portrait of the system
\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= x^2 - 1
\end{align*}
\]

Problem 3.
Show that the first order system $\dot{x} = \mu - x^3 + x$ undergoes two saddle-node bifurcations as $\mu$ varies, and find the values of $\mu$ at the bifurcation points.

Problem 4.
Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a map given by
\[
f(x) = \begin{cases} 
4x, & \text{if } x \leq \frac{1}{2}; \\
-4x + 4, & \text{if } x > \frac{1}{2}.
\end{cases}
\]
Describe the set
\[C = \{ x \in \mathbb{R} \mid \{ f^n(x) \}_{n \in \mathbb{N}} \text{ is bounded} \},\]
and find its box counting dimension.

Problem 5.
Find all fixed points of the map $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2 - 2$, and determine their stability.