LINEAR ALGEBRA, MATH 121A, HW#3

Problems

3.C.4, 3.C.6, 3.C.12,
3.D.1, 3.D.2, 3.D.7, 3.D.18, 3.D.19,
3.F.1, 3.F.7, 3.F.8, 3.F.14,

and the following problems:

<u>Problem 1.</u>

Suppose $T, S \in \mathcal{L}(V)$, T is invertible, and $TS = 0$. Prove that $S = 0$.

<u>Problem 2.</u>

Suppose $T \in \mathcal{L}(V)$ is such that $T \circ T = 0$.

a) Prove that T is not invertible;

b) Is it true that if $T \circ T = 0$, then $T = 0$? Prove or give a counterexample.

<u>Problem 3.</u>

Consider $V = \left\{ \begin{pmatrix} a & a+b \\ 0 & c \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}$.

a) Show that V is a real vector space;

b) Prove that V is isomorphic to $\mathbb{R}^3$.