Sample Final: version I

Problem 1.

Consider the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

a) Find eigenvalues of A;

b) Find eigenvectors of A.

Problem 2.

Prove that if a linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ admits a basis consisting of eigenvectors, then T^3 also admits a basis consisting of eigenvectors. Does the converse statement hold?

Problem 3.

Suppose V is a finite dimensional vector space, W_1, W_2 are subspaces, and $W_1 + W_2 = V$. Prove that

$$\dim V = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2).$$

Problem 4.

Let us denote by $M_{n\times n}(\mathbb{R})$ the set of $n \times n$ matrices with the standard operations of addition and multiplication by a real number. For each of the following statements answer whether it is true or false (explain your answer):

1) In $M_{n\times n}(\mathbb{R})$ the set of diagonal matrices is a subspace.

2) In $M_{n\times n}(\mathbb{R})$ the set of matrices of rank smaller than n is a subspace.

3) In $M_{n\times n}(\mathbb{R})$ the set of matrices of rank n is a subspace.

4) In $M_{n\times n}(\mathbb{R})$ the set of upper triangular matrices is a subspace.

Problem 5.

Let $U \subset P(\mathbb{R})$ be defined as

$$U = \text{span}(x, x + x^2, x^2 + x^3, x^3 + x^{10}, 2x + 3x^3 + 10x^{10}).$$

Find $\dim U$.