Midterm Sample

Problem 1.

Write down a basis in the space of symmetric 2×2 matrices.

Problem 2.

Let a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a reflection in the line $x = y$. Prove that T is linear and find its matrix representation (with respect to the standard basis in $\mathbb{R}^2$).

Problem 3.

Let V be a real vector space, and v_1, v_2, v_3, v_4 be a linearly independent collection of vectors.

a) Is it true that $v_1 + v_2, v_2 + v_3, v_3 + v_4$ must be linearly independent? Prove or give a counterexample.

b) a) Is it true that $v_1+v_2, v_2+v_3, v_3+v_4, v_4+v_1$ must be linearly independent? Prove or give a counterexample.

Problem 4.

Suppose $T : V \to W$ is a linear transformation, V is an infinite dimensional vector space, and W is a finite dimensional vector space. Prove that $null\ T$ must be infinite dimensional.

Problem 5.

Find nullity (dimension of the null space) and rank (dimension of range) of the linear transformation $T : M_{3\times 3} \to M_{3\times 3}$ given by $T(A) = A + A^t$.

Reminder: if $A = (a_{ij})$, then $A^t = (b_{ij})$ with $b_{ij} = a_{ji}$.