Introduction to Topology, Math 141, HW#6

Problem 1.

Show that the map $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{\pi}{2} + x - \tan^{-1}(x)$$

has no fixed points, and

$$|f(x) - f(y)| < |x - y| \text{ for all } x, y \in \mathbb{R}.$$

Why doesn't this example contradict to the Contraction Mapping Theorem?

Problem 2.

Is the set $\{\emptyset, \{0\}, \{0,1\}\}$ a topology in $\{0,1\}$?

Problem 3.

Let $(X, \mathcal{T})$ be a topological space, Y the set obtained from X by adding a single element a. Is

$$\{\{a\} \cup U \mid U \in \mathcal{T}\} \cup \{\emptyset\}$$

a topology in Y?

Problem 4.

Find out whether the following equalities hold true for any sets A and B in a topological space:

$$int(A \cap B) = int\,A \ \cap \ int\,B,$$

$$int(A \cup B) = int\,A \cup \ int\,B.$$

Problem 5.

Let X and Y be two topological spaces. Prove that a map $f : X \to Y$ is continuous if and only if the preimage of each closed set is closed.