Introduction to Topology, Math 141, HW#2

Problem 1.

Consider $S = \left\{ \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3m}} \mid n, m \in \mathbb{N} \right\} \subseteq \mathbb{R}$. Is S closed? If not, describe all points in $\bar{S} \backslash S$.

Problem 2.

Give an example of a metric space (X, d) such that any countable subset $Y \subset X$ is not dense in X.

Problem 3.

Let (X, d) be a metric space. Is it true the every one-point subset must be nowhere dense in X? Explain your answer (prove or give a counterexample).

Problem 4.

Let A, B be subsets of a metric space X. Is it true that $\operatorname{int}(A \cup B) = \operatorname{int} A \cup \operatorname{int} B$? Is it true that $\operatorname{int}(A \cap B) = \operatorname{int} A \cap \operatorname{int} B$?

Problem 5.

Let A be a subset of a metric space X. Is it true that $\overline{\operatorname{int} A} = \operatorname{int} \bar{A}$?

Problem 6.

Let (X, d) be a metric space, and $Y \subseteq X$ a dense subset such that every Cauchy sequence in Y converges in X. Prove that X is complete.

Problem 7.

Suppose that for each irrational $q \in \mathbb{R}$, an equilateral triangle T_q in the plane $\mathbb{R}^2$ is constructed such that one vertex is at $(q, 0)$ and its opposite side is above and parallel to the x-axis. Prove that $\cup_{q \in \mathbb{R} \backslash \mathbb{Q}} T_q$ must contain a rectangle of the form $[a, b] \times (0, \varepsilon) \subseteq \mathbb{R}^2$ for some $a < b$ and $\varepsilon > 0$.