INTRODUCTION TO TOPOLOGY, MATH 141, HW#6

Problem 1.

Which of the following 26 curves are homeomorphic? Give an informal explanation; you do not have to provide a formal proof in this problem.

A	B	C	D	E	F	G
H	I	J	K	L	M	N
O	P	Q	R	S	T	U
V	W	X	Y	Z		

Problem 2.

Prove that

a) the closed disc $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ is homeomorphic to the square $I = \{(x, y) \in \mathbb{R}^2 \mid x \in [0, 1], y \in [0, 1]\}$;

b) the open disc $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ is homeomorphic to the open square $\operatorname{int} I = \{(x, y) \in \mathbb{R}^2 \mid x \in (0, 1), y \in (0, 1)\}$;

c) the circle $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is homeomorphic to the boundary of the square $\partial I = I \backslash \operatorname{int} I$.

Problem 3.

A subset S of a topological space X is dense if it has non-empty intersection with any non-empty open subset of X. Prove that if $f : X \to Y$ is a surjective (i.e. "onto") continuous map, and $S \subset X$ is dense in X, then $f(S)$ is a dense subset of Y.

Problem 4.

Let X, Y be topological spaces.

a) Prove that a map $f : X \to Y$ is continuous if and only if preimage of any closed set is closed;

b) Prove that a map $f : X \to Y$ is continuous if and only if

$$\overline{f^{-1}(A)} \subset f^{-1}(\overline{A})$$

for each $A \subset Y$.

Problem 5.

Let X be a topological space, and $B \subset A \subset X$. Prove that the topology induced on B by the relative topology of A coincides with the topology induced on B directly from X.

Problem 6.

Let X be a topological space, and $f_1, f_2, \ldots, f_n : X \to \mathbb{R}$ be continuous functions. Prove that

a) the set $\{x \in X \mid f_1(x) = f_2(x) = \ldots = f_n(x) = 0\}$ is closed;

b) the set $\{x \in X \mid f_1(x) \geq 0, f_2(x) \geq 0, \ldots, f_n(x) \geq 0\}$ is closed;

c) the set $\{x \in X \mid f_1(x) > 0, f_2(x) > 0, \ldots, f_n(x) > 0\}$ is open.

Problem 7.

Let X, Y be topological spaces, and $h : X \to Y$ be a homeomorphism. Prove that for every subset $A \subset X$ the restriction $f_A : A \to f(A)$ is also a homeomorphism.