INTRODUCTION TO TOPOLOGY, MATH 141, HW#7

Problem 1.

Prove that all (infinite) arithmetic progressions consisting of positive integers form a base of a topology on $\mathbb{N}$.

Problem 2.

Prove that the collection of sets $U_a = \{(x, y) \mid x > a, y \in \mathbb{R}\}, a \in \mathbb{Q}$, form a base of topology on $\mathbb{R}^2$. Describe this topology.

Problem 3.

Prove that the collection of sets

$$U_a = \{(x, y) \mid x \in (a, b), y \in \mathbb{R}\}, a, b \in \mathbb{Q}, a < b,$$

form a base of topology on $\mathbb{R}^2$. Describe this topology. Does it coincide with the topology in Problem 2? With the standard topology on $\mathbb{R}^2$?

Problem 4.

Prove that any subspace of a topological T_3-space is also a T_3-space.

Problem 5.

Let (X, d) be a metric space, and A, B be two disjoint closed sets. Prove that there exists a continuous function $f : X \to [0, 1]$ such that $f^{-1}(0) = A$ and $f^{-1}(1) = B$.
Hint: This is much easier than Uryson's Lemma.

Problem 6.

Let us say that a continuous map is *closed* if the image of each closed set under this map is closed. Prove that a continuous map of a compact space to a Hausdorff space is closed.

Problem 7.

Let X be a topological space, and $A \subset X$ be a connected set. Prove that $\overline{A}$ is also connected.