Introduction to Topology, Math 141, Sample Midterm Exam

Problem 1.

Let X be the space of all finite subsets of $\mathbb{N}$. If $S \in X$, denote by $|S|$ the number of elements of S. Show that

$$d(S_1, S_2) = |S_1| + |S_2| - 2|S_1 \cap S_2|$$

defines a metric in X. Is (X, d) complete? Compact?

Problem 2.

For a subset S of a metric space (X, d) denote by $\overline{S}$ the closure of S, and by $\operatorname{int} S$ - the interior of S.

a) Is it true that for any subset $S \subseteq X$ we have $\operatorname{int}(\overline{\operatorname{int} S}) = \operatorname{int} S$?

b) Is it true that $\overline{\operatorname{int} \overline{S}} = \overline{S}$ for any $S \subseteq X$?

Explain your answers.

Problem 3.

Is it possible to find a sequence $\{U_n\}_{n\in\mathbb{N}}$ of open subsets of $\mathbb{R}^1$ such that $\mathbb{R}^1 = \bigcup_{n=1}^{\infty} \partial U_n$?

Problem 4.

For each of the following statements answer whether it is true or false. Explain your answers.

a) Every separable metric space is totally bounded.

b) If every subset of a metric space is open, then the metric space is complete.

c) If (X, d) is a compact metric space, $f : X \to \mathbb{R}$ is continuous, and there are points $a, b \in X$ such that $f(a) < 0$, $f(b) > 0$, then there exists a point $c \in X$ such that $f(c) = 0$.

Problem 5.

Suppose that (X, d) is a metric space, and $Y \subset X$ is a subset of X. Suppose also that $f : Y \to \mathbb{R}$ is a continuous function. Is it true that f can always be extended to a continuous function on X? In other words, is it true that there exists a continuous function $F : X \to \mathbb{R}$ such that for any $y \in Y$ we have $F(y) = f(y)$?