Problem 1.

Find the volume of the set

\[ \{(x, y, z) \mid -1 \leq z \leq 1, 4(x - \sin z)^2 + (y - \cos z)^2 \leq 1\} \]

Problem 2.

Find the area of the set

\[ \{(x, y) \mid -1 \leq x \leq 1, |y - \sqrt{1 - x^2}| \leq |x|\} \]

Problem 3.

Find \( \int_{\mathbb{R}^2} f(x, y) dxdy \), where

\[ f(x, y) = \begin{cases} 
\cos \sqrt{x^2 + y^2}, & \text{if } x^2 + y^2 \leq \frac{\pi^2}{4}; \\
0, & \text{if } x^2 + y^2 > \frac{\pi^2}{4}.
\end{cases} \]

Problem 4.

Let \( f(x, y, z) = z(x^2 + y^2) \). Find \( \int_K f \), where \( K = \{0 \leq z \leq 1, x^2 + y^2 \leq 1\} \).

Problem 5.

Find \( \int_C \omega \), where \( \omega = \sin y \cos x dx + \sin x \cos y dy \), and \( C \) is a unit circle (oriented counterclockwise).

Problem 6.

Find \( \int_C \omega \), where \( \omega = x dx + xy^2 dy \), and \( C \) is an interval connecting \((0, 0)\) and \((2, 2)\).
Problem 7.

Suppose $\alpha$ and $\beta$ are smooth 1-forms in $\mathbb{R}^2$ such that for any path $C$ we have $\int_C \alpha = \int_C \beta$. Prove that $\alpha = \beta$ (i.e. coefficients of $\alpha$ and $\beta$ are equal).

Problem 8.

Assume that $\omega$ is a 1-form in $\mathbb{R}^2$ such that $\int_C \omega = 0$ for any closed curve $C$. Prove that $\omega$ is exact.

Problem 9.

Let $D$ be a unit disc in $\mathbb{R}^2$ centered about the origin. Calculate explicitly $\int_D d\omega$ and $\int_{\partial D} \omega$ if $\omega = xdy$.

Problem 10.

TRUE OR FALSE: If $\omega$ is a $k$-form and $k$ is odd then $\omega \wedge \omega = 0$. What if $k$ is even?