DYNAMICAL SYSTEMS

Homework #3

Problem 1.

What is the number of periodic points of (not necessarily prime) period n for the solenoid map?

Problem 2.

Consider the map of the torus $F_L : \mathbb{T}^2 \to \mathbb{T}^2$ defined by the matrix

$$L = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

(i.e. $F_L(x, y) = (2x + 3y, x + 2y)(\mod 1)$). Find the number of periodic points of the map F_L of each period $n \in \mathbb{N}$.

Problem 3.

Consider the following map $F : D \times S^1 \to D \times S^1$, where D is a disc $\{x^2 + y^2 < 1\}$ and S^1 is a circle, given by $F(x, y, \phi) = (0.1x, 0.1y, \phi)$. The maximal attractor of this map is a circle $\{x = y = 0\}$. Prove that if G is C^1-close to F then its maximal attractor is also a C^1-smooth closed curve (it is enough to provide a sketch of the proof, without all the technical details).

Problem 4.

This problem will not be graded. Suggest (as many as you can, better at least three) problems on the topics covered (expanding maps of a circle, topological Markov chains, hyperbolic automorphism of a torus) that you would suggest for this homework. You do not need to provide solutions.