# ERGODIC THEORY

## Homework #1

### Problem 1.

Let $T : (X, \mathfrak{B}, \mu) \to (X, \mathfrak{B}, \mu)$ be an ergodic MPT in a finite measure space, $A \in \mathfrak{B}$, $\mu(A) > 0$. Prove that the first return map $T_A : A \to A$ is also measure preserving and ergodic.

### Problem 2.

Prove that a measure-preserving transformation $T : X \to X$ of a probability space $(X, \mu)$ is ergodic iff any measurable function $f : X \to \mathbb{R}$ that increases along the orbits almost everywhere (i.e. $f(T(x)) \geq f(x)$ for $\mu$-a.e. $x$) is a constant a.e.

### Problem 3.

Let $X$ be a compact separable metric space, $\mu$ a Borel probability measure, $T : X \to X$ a continuous MPT. Suppose also that measure of any open set is positive, and that $T$ is (strongly) mixing. Prove that $T : X \to X$ is topologically mixing.