COMPLEX ANALYSIS MATH 220A

Final Exam (Sample)

Problem 1.

Let f be a meromorphic function on $\mathbb{C}$ which is analytic in a neighborhood of 0. Suppose that the power series of f at 0 has only real and positive coefficients, and its radius of convergence is equal to one. Prove that f has a pole at $z = 1$.

Problem 2.

Evaluate the integral

$$\int_{\partial D(0,2)} (z-1)e^{\frac{z}{z-1}}\,dz.$$

Problem 3.

Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos^3 x}{a^2+x^2}dx$$

where $a \in \mathbb{R}$, $a > 0$.

Problem 4.

Suppose that f is an entire function that has the form

$$f(x+iy) = e^x(g(y) + ih(y)),$$

where g, h are C^∞ real-valued functions of real argument. Suppose also that $g(0) = 1$ and $h(0) = 0$. Find the explicit form of f. Is it uniquely determined by the given conditions?

Problem 5.

Let n be a positive integer. Prove that the polynomial

$$f(x) = \sum_{i=1}^{n} \frac{x^i}{i!} = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!}$$

has n distinct complex zeros, $z_1, z_2, \ldots, z_n$, and they satisfy

$$\sum_{i=1}^{n} z_i^{-k} = 0 \;\text{ for }\; 2 \le k \le n.$$