Complex Analysis Math 220B

Final Exam

This is a take home open book exam. Please, choose 2 hours of your time, and work on this exam at home. You should time it, and stop working on the exam after 2 hours. You can use the textbook and/or your notes, but you cannot use any online sources or help of other people. After that please scan the exam and send it to the instructor (Anton Gorodetski) via email (asgor@uci.edu) as an attachment. The deadline for the submission is 11am on Wednesday, March 18, 2020.

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

For each of the following statements explain whether it is TRUE or FALSE (i.e. prove or give a counterexample):

a) Any sequence of polynomials restricted to the unit disc $\mathbb{D} = \{|z| < 1\}$ has a normally convergent subsequence;

b) Any sequence of polynomials restricted to the unit disc $\mathbb{D}$ has a normally convergent subsequence if there is a uniform upper bound on their degrees;

c) Any sequence of polynomials restricted to the unit disc $\mathbb{D}$ has a normally convergent subsequence if there is a uniform upper bound on their values in the unit disc;

d) If a sequence of polynomials uniformly converges on the unit disc $\mathbb{D}$, then there is a uniform upper bound on their degrees.

Reminder: normal convergence on $\mathbb{D}$ $\Leftrightarrow$ uniform convergence on compact subsets of $\mathbb{D}$ to a function (not to ∞)

Problem 2.

Suppose $\{u_i\}_{i \in \mathbb{N}}$ is a sequence of harmonic functions on $\mathbb{C}$ such that for any $i, j \in \mathbb{N}$ either $u_i(z) > u_j(z)$ for all $z \in \mathbb{C}$, or $u_i(z) < u_j(z)$ for all $z \in \mathbb{C}$. Prove that if there exists a finite limit $\lim_{i \to \infty} u_i(0)$, then the sequence of functions $\{u_i\}_{i \in \mathbb{N}}$ converges uniformly on $\mathbb{C}$.

Problem 3.

Find the largest open set $U \subset \mathbb{C}$ where the product

$$\prod_{n=1}^{\infty}\left(1-\frac{1}{n^2}\exp\left(\frac{nz}{z-1}\right)\right)$$

converges normally to a holomorphic function.

Problem 4.

Let $r < 1 < R$. Prove that for all sufficiently small $\varepsilon > 0$ the polynomial

$$p(z) = \varepsilon z^{10} + z^5 + 1$$

has exactly five roots (counted with their multiplicities) inside the annulus

$$r\varepsilon^{-\frac{1}{5}} < |z| < R\varepsilon^{-\frac{1}{5}}.$$

Problem 5.

Is there an analytic function f on $\mathbb{D} = \{|z| < 1\}$ such that $|f(z)| < 1$ for $|z| < 1$, $f(0) = \frac{1}{2}$, and $f'(0) = \frac{3}{4}$? If so, find such an f. Is it unique?