COMPLEX ANALYSIS, HW # 4

Chapter 7, problems 23, 28, 38, 39, **and these problems:**

Problem 1.

TRUE or FALSE: There exists a bounded harmonic function on the upper half plane $\mathbb{H}$ that cannot be extended to any larger domain. Explain your answer.

Problem 2.

Set $U = \{z = x + iy \mid y > 0, -1 < x < 1\}$. Suppose $f : \bar{U} \to \mathbb{C}$ is continuous and holomorphic in U. Suppose also that for any real $x \in (-1, 1)$ we have $f(x) = x^{2020}$. Prove that $f(z) = z^{2020}$ for all $z \in U$.

Problem 3.

Suppose a continuous function $u : \mathbb{C} \to \mathbb{R}$ has the following property:

$$u(x + iy) = \frac{1}{4}(u(x + a + iy) + u(x - a + iy) + u(x + i(y + a)) + u(x + i(y - a)))$$

for all $a \in \mathbb{C}$. Does it imply that u is harmonic?