COMPLEX ANALYSIS, HW # 5

Chapter 7, problems 42, 49, 50; Chapter 8, problems 3, 5, and these problems:

Problem 1.

Suppose $u : U \to \mathbb{R}$ is a non-constant harmonic function on a connected open set. Prove that the set of points, where gradient of u vanishes, consists of isolated points.

Problem 2.

Is there a harmonic function $u : \mathbb{D} \to \mathbb{R}$ such that $\{z \mid u(z) = 0\}$ is an interval $[0, 1) \subset \mathbb{D}$? Give an example or prove that such function does not exist.