COMPLEX ANALYSIS MATH 220B

Final Exam (sample)

Problem 1.

Prove that the product

$$\prod_{n=1}^{\infty}\left(\sin\left(\frac{z}{(2020)^n}\right)+\exp\left(\frac{z^{2020}}{n!}\right)\right)$$

converges uniformly on compact sets to an entire function.

Problem 2.

Let u be a real valued harmonic function in the complex plane such that

$$u(z)\leq a|\log|z||+b,$$

for all z, where a and b are positive constants. Prove that u is constant.

Problem 3.

Find the number of solutions of the equation $2z^8+16z+15=0$ in the left-half plane $\{z\mid \operatorname{Re} z<0\}$.

Problem 4.

Let f be analytic on the upper-half plane and satisfy $f(z)<1$. Furthermore suppose $f(i)=0$. Give an upper bound for $f'(i)$ and state which functions realize this extremum.

Problem 5.

Let $D=\{z\in\mathbb{C}\mid |z|\leq 1,\ z\notin\{-1,1\}\}$. Find an explicit function $f:D\to\mathbb{R}$ such that the following conditions are satisfied:

- f is harmonic on the interior of D (the open unit disc);
- $f(z)=1$ when $|z|=1$ and $\operatorname{Im} z>0$;
- $f(z)=-1$ when $|z|=1$ and $\operatorname{Im} z<0$.