

COMPLEX ANALYSIS MATH 220B

Midterm Exam (sample)

Problem 1.

How many roots of the equation $z^4 + z^3 - 4z + 1 = 0$ are in the ring $1 < |z| < 4$?

Problem 2.

Find a conformal mapping of the domain U to the unit disc, where

$$U = \{|z| > 1, z \notin I\}, \quad I = [1, 2] \subset \mathbb{R}.$$

Problem 3.

Let $\mathbb{H} = \{z \in \mathbb{C} : \text{Im } z > 0\}$ and $f : \mathbb{H} \rightarrow \mathbb{H}$ be analytic. Prove that

$$\left| \frac{f(z) - f(i)}{f(z) - \overline{f(i)}} \right| \leq \left| \frac{z - i}{z + i} \right|, \quad z \in \mathbb{H}.$$

Problem 4.

Let $f(z)$ be holomorphic in the unit disc \mathbb{D} and continuous on the closed disc $\overline{\mathbb{D}}$. Suppose $f(e^{i\theta}) = e^{ie^{i\theta}}$ for $0 < \theta < \frac{\pi}{4}$. Prove $f(z) \equiv e^{iz}$ on \mathbb{D} .

Problem 5.

Let \mathbb{D} be the unit disc, and suppose that $f : \mathbb{D} \rightarrow \mathbb{D} \setminus \{0\}$ is analytic and $f(0) = \frac{1}{2}$. Prove that $|f(\frac{1}{2})| \geq \frac{1}{8}$.