COMPLEX ANALYSIS MATH 220B

Final Exam (Sample)

Problem 1.

Show that the infinite product
$$\prod_{n=1}^{\infty}\left(1-\frac{z}{\sqrt{n}}\right)e^{\frac{z}{\sqrt{n}}}$$
converges to a holomorphic function. Find all zeros of this holomorphic function.

Problem 2.

TRUE or FALSE: There exists a bounded harmonic function on the upper half plane $\mathbb{H}$ that cannot be extended to any larger domain. Explain your answer.

Problem 3.

Let f be analytic function in the unit disc $\mathbb{D}$. Prove that there exists a sequence $\{z_n\} \subset \mathbb{D}$ such that $|z_n| \to 1$ as $n \to \infty$, and $\{f(z_n)\}$ is bounded.

Problem 4.

Prove that the family $\mathcal{F}$ of functions holomorphic in the unit disc with power series $f(z) = \sum_{n=1}^{\infty} a_n z^n$ that satisfy $|a_n| \leq n^{2018}$ is normal.

Problem 5.

TRUE OR FALSE: If $u : \overline{\mathbb{D}} \to \mathbb{R}$ is a continuous function subharmonic in $\mathbb{D}$, then $\sin u$ is also subharmonic in $\mathbb{D}$. Prove or give a counterexample.