COMPLEX ANALYSIS, HW # 3

Chapter 6, problems 20, 22, 23, 24, and these problems:

Problem 1.

Let $U \neq \mathbb{C}$ be a simply connected domain in $\mathbb{C}$. Let $f : U \to U$ be a conformal mapping that fixes two distinct points in U (i.e. there are $p, q \in U$ such that $f(p) = p$ and $f(q) = q$). Show that $f(z) \equiv z$.

Problem 2.

Let $\{f_\alpha\}_{\alpha \in A}$ be a family of holomorphic functions on the unit disc D such that

$$\forall z \in D \ \ \forall f \in \{f_\alpha\} \quad \mathbf{Re} f(z) \neq \mathbf{Im} f(z).$$

Prove that $\{f_\alpha\}_{\alpha \in A}$ is a normal family.

Problem 3.

Show that there is a holomorphic function defined in the set

$$\Omega = \{z \in \mathbb{C} \mid |z| > 4\}$$

whose derivative is

$$\frac{z}{(z-1)(z-2)(z-3)}.$$

Is there a holomorphic function on Ω whose derivative is

$$\frac{z^2}{(z-1)(z-2)(z-3)}?$$