COMPLEX ANALYSIS, HW # 4

Chapter 7, problems 30, 31, 38, 39, and these problems:

Problem 1.

Let $f_n : D(0,1) \to D(0,1)\backslash\{0\}$ be holomorphic functions with $f_n(0) = \frac{1}{n}$ $(n = 1, 2, 3, \cdots.)$ Prove $\sum_{n=1}^{\infty} f_n(z)^3$ converges uniformly and absolutely on any compact subset K in $D(0, 1/2)$.

Problem 2.

Set $U = \{z = x + iy \mid y > 0, -1 < x < 1\}$. Suppose $f : \bar{U} \to \mathbb{C}$ is continuous and holomorphic in U. Suppose also that for any real $x \in (-1, 1)$ we have $f(x) = x^{2018}$. Prove that $f(z) = z^{2018}$ for all $z \in U$.

Problem 3.

Let $I \subset \mathbb{R} \subset \mathbb{C}$ be the closed interval $[0, 1/2]$.

a) Construct a conformal mapping of $\mathbb{C}\backslash I$ to $\mathbb{D}\backslash\{0\}$.

b) Prove that $\mathbb{D}\backslash I$ is not conformally equivalent to $\mathbb{D}\backslash\{0\}$.