Complex Analysis, HW # 6

Chapter 7, problems 41, 48, 50, 69 (in all these problems assume that the functions are continuous), and these problems:

Problem 1.

Let $\mathcal{F}$ be a family of analytic functions on Δ for which there exists $M > 0$ such that

$$\int_{\Delta} |f(z)| dx\, dy \leq M \text{ for all } f \in \mathcal{F}.$$

Show that $\mathcal{F}$ is a normal family.

Problem 2.

Let f and g be analytic on a bounded domain D and continuous on its closure. Show that $|f(z)| + |g(z)|$ attains its maximum on the boundary of D.

Problem 3.

Let $f(z)$ be holomorphic in the unit disc $\mathbb{D}$ and continuous on the closed disc $\overline{\mathbb{D}}$. Suppose $f(e^{i\theta}) = e^{ie^{i\theta}}$ for $0 < \theta < \frac{\pi}{4}$. Prove $f(z) \equiv e^{iz}$ on $\mathbb{D}$.