COMPLEX ANALYSIS MATH 220B

Midterm Exam (Sample)

Problem 1.

Describe those polynomials $a + bx + cy + dx^2 + exy + fy^2$ with real coefficients that are the real parts of entire functions.

Problem 2.

Find explicitly a conformal mapping of the domain

$$\{z \in \mathbb{C} \mid \ |z| > 1, \ \ \mathrm{Re}\, z > \mathrm{Im}\, z\}$$

to the unit disc.

Problem 3.

TRUE or FALSE: An open connected domain $U \subset \mathbb{C}$ is simply connected if and only if for any non-vanishing holomorphic function $f : U \to \mathbb{C}$ there exists a holomorphic function $g : U \to \mathbb{C}$ such that $(g(z))^{2018} = f(z)$ for all $z \in U$. Explain your answer.

Problem 4.

Let U be a bounded, connected, open subset of $\mathbb{C}$, and let f be a non-constant continuous function on $\overline{U}$ which is holomorphic on U. Assume that $|f(z)| = 1$ for z on the boundary of U.

(a) Show that 0 is in the range of f.

(b) Show that f maps U onto the unit disc.

Problem 5.

Suppose a continuous function $u : \mathbb{C} \to \mathbb{R}$ has the following property:

$$u(x+iy) = \frac{1}{4}(u(x+a+iy) + u(x-a+iy) + u(x+i(y+a)) + u(x+i(y-a)))$$

for all $a \in \mathbb{C}$. Does it imply that u is harmonic?