COMPLEX ANALYSIS, HW # 4

Section 10, problems 2, 3, 4, 17 a)-d) – describe the corresponding Riemann surfaces (in particular, determine their genus), and the following problems:

Problem 1.

Let

$$U_1 = \{z \in \mathbb{C} \mid \operatorname{Im} z > 0,\ |z| > 1\}, \quad U_2 = \{z \in \mathbb{C} \mid \operatorname{Im} z < 0,\ |z| < 1\}.$$

Find explicitly conformal mappings of U_1 to $\mathbb{H} = \{\operatorname{Im} z > 0\}$ and of U_2 to $\mathbb{H}$.

Problem 2.

Let $\zeta(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$ be Joukowski function. Prove that $\zeta(U_1) = \mathbb{H}$ and $\zeta(U_2) = \mathbb{H}$, where U_1, U_2 are defined in the previous problem.

Problem 3.

Find explicitly a conformal mapping of

$$U = \{z \in \mathbb{C} \mid \operatorname{Re} z > 1 \text{ and } (\operatorname{Im} z > 0 \text{ or } |z-1| < 1)\}$$

to the upper half plane.