Problem 1.

Determine the number of zeros of the polynomial
\[ z^{100} + 50z^{50} + 100z^2 + 1 \]
in the annulus \( \{1 < |z| < 2\} \).

Problem 2.

Let \( U \) be a bounded open connected set, \( \{f_n\} \) a sequence of continuous functions on the closure of \( U \), analytic in \( U \). Assume that \( \{f_n\} \) converges uniformly on \( \partial U \). Prove that \( \{f_n\} \) converges uniformly on \( U \).

Problem 3.

Find an explicit conformal map of the open set
\[ \{0 < \text{Im } z < 1\} \setminus \{z = it, \ t \in [0, 1/2]\} \]
to the unit disc.

Problem 4.

Find the integral
\[ \int_0^\pi \frac{d\theta}{3 + 2\cos \theta}. \]

Problem 5.

Let \( \mathcal{F} \) be the family of all analytic functions
\[ f(z) = z + a_2z^2 + a_3z^3 + \ldots \]
on the open unit disc, such that \( |a_n| \leq n \) for each \( n \). Prove that \( \mathcal{F} \) is a normal family.