COMPLEX ANALYSIS MATH 220 C

Midterm Exam (sample)

Problem 1.

What is the image of the upper half-plane under a mapping of the form

$$h(z) = \frac{az+b}{cz+d}, \quad a, b, c, d \text{ are real}, \quad ad - bc < 0 \ ?$$

Problem 2.

Show that

$$1 - \frac{1}{2^z} + \frac{1}{3^z} - \frac{1}{4^z} + \frac{1}{5^z} - \dots$$

can be continued analytically to the full plane (i.e. show that its complete analytic continuation is an entire function).

Problem 3.

Suppose $\{n_k\}_{k=1,2,\dots}$ is an increasing sequence of positive integers such that the infinite series $\sum_{k=1}^{\infty} \frac{1}{n_k}$ diverges. Prove that if f is a bounded holomorphic function on $\{\mathrm{Re}\ z\}$ (the right-hand half-plane) having a zero at each n_k, then f must be identically equal to zero.

Problem 4.

TRUE or FALSE: There exist a holomorphic function f on $\{\ |z| < 1\ \}$ with the property that for every sequence $\{z_n\}$ of points in the unit disk for which $|z_n| \to 0$ as $n \to \infty$, the corresponding image sequence $\{f(z_n)\}_{n=1,2,\dots}$ is an unbounded subset of $\mathbb{C}$?

Problem 5.

Prove that the product $\prod_{k=1}^{\infty} \left(\frac{z^n}{n!} + \exp\left(\frac{z}{2^n}\right) \right)$ converges uniformly on compact sets to an entire function.