

COMPLEX ANALYSIS, HW # 2

Chapter 10, problem 16, and these problems:

Problem 1.

Consider a subgroup of $Aut(\mathbb{C})$ generated by the maps $f(z) = z + 1, g(z) = z + i, h(z) = -z$. Describe its fundamental domain.

Problem 2.

Consider a subgroup of $Aut(\mathbb{C} \setminus \{0\})$ generated by the maps $f(z) = iz, g(z) = 2z$. Describe its fundamental domain. What is a genus of the Riemann surface that one gets factorizing $\mathbb{C} \setminus \{0\}$ by the action of $\langle f, g \rangle$?

Problem 3.

Prove that any meromorphic function on the Riemann sphere must be a rational function in z .

Problem 4.

Show that genus of $\{(z, w) \in \mathbb{C}^2 \mid z^n + w^n = 1\}$ is equal to $\frac{(n-1)(n-2)}{2}$.

Problem 5.

Show that the range of the entire function $\frac{\sin z}{z}$ is the whole complex plane \mathbb{C} .

Problem 6.

Let f be a non-constant, entire function such that $f(1 - z) = 1 - f(z)$. Determine the image of f .