COMPLEX ANALYSIS, HW # 3

Chapter 10, problems **18, 19, 20, 21,** and these problems:

Problem 1.

We proved that Weierstrass $\wp$-function (doubly periodic with periods 1 and i) satisfies the differential equation
$$(\wp'(z))^2 = 4(\wp(z))^3 - C_1\wp(z) + C_2.$$
Show that $C_1 = 60 \sum_{(m,n)\neq(0,0)} \frac{1}{(m+in)^4}$.

Problem 2.

Find $\zeta(2)$.
Hint: This problem is known as the Basel Problem; you can present any proof, not necessarily using complex analysis.

Problem 3.

The Dirichlet η-function (alternating zeta function) is defined as $\eta(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^z}$. Find a closed form expression for $\eta(z)$ using ζ-function.