Complex Analysis, HW # 4

Chapter 15, problems 2, 3, 6, and these problems:

A continuous random variable X is said to have a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if it has a probability density function (pdf) given by

$$f(x, \alpha, \beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & \text{if } x > 0; \\ 0, & \text{otherwise.} \end{cases}$$

Problem 1.

Check that $f(x, \alpha, \beta)$ is indeed a probability density function.

Problem 2.

Check that if X has the gamma distribution with parameters $\alpha > 0$ and $\beta > 0$, then $\mathbb{E}X = \alpha\beta$.

Problem 3.

Check that if X has the gamma distribution with parameters $\alpha > 0$ and $\beta > 0$, then $Var(X) = \alpha\beta^2$.

Problem 4.

Prove Gauss formula:

$$\Gamma(x)\Gamma\left(x + \frac{1}{n}\right)\Gamma\left(x + \frac{2}{n}\right)\Gamma\left(x + \frac{3}{n}\right) \cdot \ldots \cdot \Gamma\left(x + \frac{n-1}{n}\right) = (2\pi)^{(n-1)/2}\, n^{1/2-nx}\Gamma(nx).$$