Complex Analysis, HW # 6

Problem 1.

Show that any polynomial $f(z) = az^2 + bz + d$, $a \neq 0$, is conjugate to a map $f_c(z) = z^2 + c$ for some $c \in \mathbb{C}$.

Problem 2.

Let $f(z) = z^d + a_1 z^{d-1} + \ldots + a_d$. Show that is $f(w) = 0$, then $|w| \leq 2 \max_j |a_j|^{1/j}$.

Problem 3.

Suppose $f(z)$ is a polynomial, $deg\ f \geq 2$. Let Ω_∞ be the basis of attraction of ∞ (which is a superattracting fixed point). Show that

$$F(f) = \Omega_\infty \cup \left(\widehat{\mathbb{C}} \backslash \overline{\Omega_\infty} \right).$$

Problem 4.

Let $\{p_0, p_1, \ldots, p_{m-1}\}$, $m \geq 2$, be an attracting periodic orbit of a rational map f. Prove that all these points belong to different connected components of $F(f)$.

Problem 5.

Let g be an automorphism of the unit disc $\mathbb{D}$. Assume that g has no fixed points in $\mathbb{D}$. Prove that g^n converge (uniformly on compact sets) to a constant function. *Hint: g can be extended continuously to $\overline{\mathbb{D}}$, and any continuous map of the closed disc to itself must have at least one fixed point.*

Problem 6.

Let $g : \mathbb{D} \to \mathbb{D}$ be a holomorphic map. Assume that g admits a fixed point a in $\mathbb{D}$ which is not attractive. Show that g is an automorphism of $\mathbb{D}$.

Problem 7.

TRUE or FALSE: If $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ is a rational map, $deg\, f \geq 2$, then for any $z \in J(f)$ its orbit $\{z, f(z), f^2(z), \ldots\}$ is dense in $J(f)$.