Linear Algebra
Math 6G, Summer 2012

Practice Final Exam

Problem 1.

Find a basis in the linear subspace in $\mathbb{R}^4$ spanned by vectors $[1, 2, 3, 4]$, $[4, 3, 2, 1]$, $[1, 1, 1, 1]$, and $[-2, 0, 2, 4]$.

Problem 2.

Find the coordinate vector of the polynomial $p(x) = x^2 + x^3$ relative to the ordered basis $B = ((x-1)^3, (x-1)^2, (x-1), 1)$ of the vector space P_3.

Problem 3.

Suppose $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ and $det\ A \neq 0$. Find the unique solution of the system $A\bar{x} = \bar{w}$, where $\bar{w} = \begin{bmatrix} 2c_1 \\ 2c_2 \\ 2c_3 \end{bmatrix}$.

Problem 4.

Find $det\ A$, where

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 0 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Problem 5.

Find the area of the triangle in $\mathbb{R}^3$ with vertices $(1, 1, 1)$, $(1, 2, 3)$, and $(-5, 0, 2)$.

Problem 6.

Let $T : P_3 \to P_3$ be the linear transformation defined by $T(p(x)) = p(x) + \frac{d}{dx}p(x)$. Find the matrix representation of T relative to basis $B = (1, x, x^2, x^3)$.

Problem 7.

TRUE or FALSE: For any value of the parameter $\alpha \in \mathbb{R}$ the matrix $\begin{bmatrix} \alpha & 1 \\ 1 & 0 \end{bmatrix}$ is diagonalizable. Justify your answer.

Problem 8.

Linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by

$$T([x_1, x_2, x_3]) = [-3x_1 + 10x_2 - 6x_3, 7x_2 - 6x_3, x_3].$$

Find eigenvalues and eigenvectors of T.