COMPLEX ANALYSIS MATH 220A

Final Exam

Monday, December 7, 2009 — 1:30 - 3:30 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Show that there is a holomorphic function defined in the set

$$\Omega = \{z \in \mathbb{C} \mid |z| > 4\}$$

whose derivative is

$$\frac{z}{(z-1)(z-2)(z-3)}.$$

Is there a holomorphic function on Ω whose derivative is

$$\frac{z^2}{(z-1)(z-2)(z-3)}?$$

Problem 2.

Evaluate the improper integral

$$\int_{-\infty}^{+\infty} \frac{x^2 \sin(\pi x)}{x^3 - 1} dx$$

Problem 2 (continuation)

Problem 3.

Let $f(z)$ be analytic on $\mathbb{C}\backslash\{1\}$ and have a simple pole at $z = 1$ with residue λ. Prove that for every $R > 0$,

$$\lim_{n\to\infty} R^n \left| (-1)^n \frac{f^{(n)}(2)}{n!} - \lambda \right| = 0.$$

Problem 4.

Find the number of zeros of the function $f(z) = 2z^5 + 8z - 1$ in the annulus $1 < |z| < 2$.

Problem 5.

Let f be an entire function. Suppose that for each complex number a, the power series expansion

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n, \quad z \in \mathbb{C},$$

has at least one coefficient $c_n = 0$. Show that f is a polynomial.