Complex Analysis Math 220A

Midterm Exam

Friday, October 30, 2009 — 12:00 pm - 1:00 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find the radius of convergence for the series:

$$\sum_{n=1}^{+\infty} \frac{z^{2n}}{n!} \quad \text{and} \quad \sum_{n=1}^{+\infty} \frac{z^{n!}}{2n}$$

Problem 2.

Find all entire functions $f(z)$ on $\mathbb{C}$ satisfying

$$|f(z)| \leq |z|e^x, \quad z = x + iy \in \mathbb{C}.$$

Problem 3.

Let f be a non-constant entire function. Prove that if $\lim_{|z|\to+\infty} |f(z)| = +\infty$ then f must be a polynomial.

Problem 4.

Show that for any $R > 0$, there is N_R such that when $n > N_R$, the function

$$P_n(z) = 1 + z + \frac{z^2}{2!} + \ldots + \frac{z^n}{n!} \neq 0 \quad \text{for all} \quad |z| \leq R.$$

Problem 5.

Let $p(z)$ be a polynomial. Suppose that $p(z) \neq 0$ for $\mathrm{Re}(z) > 0$. Prove that $p'(z) \neq 0$ for $\mathrm{Re}(z) > 0$.