Solution of Homework 6

Problem (4.38):

Solution:

By residue theorem,

$$\frac{1}{2\pi i}\oint_{\partial D(P,r)}\frac{f(z)}{g(z)}dz=\sum_{i=1}^{k}Res(\frac{f}{g};z=P_i)$$

If all P_i are simple zeros, we have

$$\frac{1}{2\pi i}\oint_{\partial D(P,r)}\frac{f(z)}{g(z)}dz=\sum_{i=1}^{k}Res(\frac{f}{g};z=P_i)=\sum_{i=1}^{k}\frac{f(P_i)}{g'(P_i)}$$

If the zeros are not simple, with orders $\alpha_1,\alpha_2,\cdots,\alpha_k$, then

$$\frac{1}{2\pi i}\oint_{\partial D(P,r)}\frac{f(z)}{g(z)}dz=\sum_{i=1}^{k}Res(\frac{f}{g};z=P_i)=\sum_{i=1}^{k}\frac{[\frac{f(z)}{g(z)}(z-P_i)^{\alpha_i}]^{(\alpha_i-1)}}{(\alpha_i-1)!}|_{z=P_i}$$

■

Problem (4.46):

Solution:

$$\int_0^\infty\frac{1}{1+x^4}dx=\frac{1}{2\sqrt{2}}\pi$$

■

Problem (4.47)

Solution:

$$\int_{-\infty}^{\infty} \frac{\cos x}{1+x^4} dx = \frac{\pi}{\sqrt{2}e^{\frac{1}{\sqrt{2}}}}(\sin(\frac{1}{\sqrt{2}}) + \cos(\frac{1}{\sqrt{2}}))$$

■

Problem (4.51)

Solution:

$$\int_{0}^{\infty} \frac{x \sin x}{1+x^2} dx = \frac{\pi}{2e}$$

■

Problem (4.53)

Solution:

$$\int_{-\infty}^{\infty} \frac{x}{\sinh x} dx = \frac{\pi^2}{2}$$

■

Problem (4.55)

Solution:

$$\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx = \pi$$

■

Problem (4.58)

Solution:

$$\int_{-\pi}^{\pi} \frac{1}{5+3\cos\theta} d\theta = \frac{\pi}{2}$$

■