Solution of Homework 7

Problem (4.48):

Solution:

$$\int_0^{+\infty} \frac{x^{\frac{1}{4}}}{1+x^3} dx = \frac{\pi}{3\sin(\frac{5\pi}{12})}$$

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Problem (4.50):

$$\int_0^{+\infty} \frac{1}{1+x^3} dx = \frac{\pi}{3\sin(\frac{\pi}{3})} = \frac{2\sqrt{3}\pi}{9}$$

Solution:

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Problem (4.54)

Solution:

$$\int_{-\infty}^{0} \frac{x^{\frac{1}{3}}}{1+x^5} dx = \frac{\pi}{5\sin(\frac{4\pi}{15})}$$

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Problem (4.56)

Solution:

$$\int_{-\infty}^{+\infty} \frac{x^4}{1+x^{10}} dx = \frac{\pi}{5}$$

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Problem (5.2)

Solution:

suppose f has zeros at $P_1, P_2, ..., P_k$ with order $\lambda_1, \lambda_2, ..., \lambda_k$.

then

$$\frac{1}{2\pi i}\oint_\gamma \frac{f'(z)}{f(z)} \cdot g(z)dz = \sum_{i=1}^{k} Res(\frac{f'(z)}{f(z)} \cdot g(z), P_i) = \sum_{i=1}^{k} \lambda_i g(P_i)$$

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Problem (5.5)

Solution:

For example, let $f_j(z) = (z - \frac{1}{2})^\iota(z - 1 + \frac{1}{j})^{k-\iota}$, then $f_j(z)$ goes to $f(z) = (z - \frac{1}{2})^\iota(z - 1)^{k-\iota}$ as $j \to \infty$. $f_j(z)$ has k roots in $D(0, 1)$, but $f(z)$ only has exactly ι roots in $D(0, 1)$.

We need to assume that f has no zeros on $\partial D(0, 1)$, then we can guarantee that f does have at least k roots. ■

Problem (5.8) **Solution:**

Since $f(z) \neq 0$ on $\partial D(P, r)$ and $\partial D(P, r)$ is compact, $|f(z)| \geq \varepsilon$ on $\partial D(P, r)$ for some $\varepsilon > 0$. Suppose $|f(z) - g(z)| < \varepsilon$ for all $z \in \partial D(P, r)$, then

$$|f(z) - g(z)| < |f(z)| + |g(z)|$$

By Rouche's theorem, f and g have the same number of zeros in $D(P, r)$ counting multiplicity.

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