COMPLEX ANALYSIS MATH 220A

Midterm Sample Exam

Problem 1.

Find the largest disk centered at 1 in which the Taylor series for

$$\frac{1}{1+z^2} = \sum a_k (z-1)^k$$

will converge.

Problem 2.

Let $f(z)$ be entire holomorphic function on $\mathbf{C}$ such that $|f(z)| \leq |\cos z|$. Prove $f(z) = c\cos z$ for some constant c.

Problem 3.

Prove that there is no entire analytic function such that

$$\bigcup_{n=0}^{\infty} \left\{ z \in \mathbb{C} \,:\, f^{(n)}(z) = 0 \right\} = \mathbb{R}.$$

Problem 4.

Is there an entire function f such that

$$f\left(\frac{1}{n}\right) = f\left(-\frac{1}{n}\right) = \frac{1}{n^3}$$

for all $n \in \mathbb{N}$? Justify your answer.

Problem 5.

Find the radius of convergence R_1 of the series

$$\sum_{n=1}^{+\infty} \frac{z^n}{n^2}$$

and show the series converges uniformly on $\overline{D(0,R_1)}$. What is the radius of convergence R_2 of the derivative of this series? Does it converge uniformly on $\overline{D(0,R_2)}$?