COMPLEX ANALYSIS MATH 220B

Final Sample Exam

Problem 1.

Prove that the product $\prod_{k=1}^{\infty} \left(\frac{z^n}{n!} + \exp\left(\frac{z}{2^n}\right)\right)$ converges uniformly on compact sets to an entire function.

Problem 2.

Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function such that

$$f(z+1) = f(z), \quad \text{and} \quad |f(z)| \le e^{|z|},\ z \in \mathbb{C}.$$

Prove that f must be a constant.

Problem 3.

Let $h : \mathbb{C} \to \mathbb{R}$ be harmonic and non-constant. Show that $h(\mathbb{C}) = \mathbb{R}$.

Problem 4.

Let f be holomorphic in $D(0,2)$ and continuous in $\overline{D(0,2)}$. Suppose that $|f(z)| \le 16$ for $z \in D(0,2)$ and $|f(0)| = 1$. Prove that f has at most 4 zeros in $D(0,1)$.

Problem 5.

Denote $A = \{r < |z| < R\},\ 0 < r < R < \infty$. TRUE OR FALSE: For every $\varepsilon > 0$ there exists a polynomial $p(z)$ such that

$$\sup\left\{ \left| p(z) - \frac{1}{z^2} \right|,\ z \in A \right\} < \varepsilon.$$