COMPLEX ANALYSIS MATH 220B

Midterm Sample Exam

Problem 1.

Let $L \subset \mathbb{C}$ be the line $L = \{z = x+iy \mid x = y\}$. Assume that $f : \mathbb{C} \to \mathbb{C}$ is an entire function such that for any $z \in L$ we have $f(z) \in L$. Assume that $f(1) = 0$. Prove that $f(i) = 0$.

Problem 2.

Let u be a harmonic function on $\mathbb{R}^2$ that does not take zero value (i.e. $u(x) \neq 0 \ \ \forall x \in \mathbb{R}^2$). Show that u is constant.

Problem 3.

Find explicitly a conformal mapping of the domain

$$\{z \in \mathbb{C} \mid |z| < 1, \ \ \mathrm{Re}\, z > 0, \ \mathrm{Im}\, z > 0\}$$

to the unit disc.

Problem 4.

a) Suppose a continuous function $u : \mathbb{C} \to \mathbb{R}$ has the following property:

$$u(x+iy) = \frac{1}{4}(u(x+a+iy) + u(x-a+iy) + u(x+i(y+a)) + u(x+i(y-a)))$$

for all $a \in \mathbb{R}$. Does it imply that u is harmonic?

b) Suppose a continuous function $u : \mathbb{C} \to \mathbb{R}$ has the following property:

$$u(x+iy) = \frac{1}{4}(u(x+a+iy) + u(x-a+iy) + u(x+i(y+a)) + u(x+i(y-a)))$$

for all $a \in \mathbb{C}$. Does it imply that u is harmonic?

Problem 5.

Describe those polynomials $a + bx + cy + dx^2 + exy + fy^2$ with real coefficients that are the real parts of analytic functions on $\mathbb{C}$.