ERGODIC THEORY, HW # 4

Homework will not be collected or graded. Nevertheless, please, make sure that you understand how to solve the problems. Interaction between students is strongly encouraged.

Problem 1.

Show that for every $s > 0$ there exists some Bernoulli shift whose entropy is equal to s.

Problem 2.

Let (M, μ) be a measure space with probability measure μ. For two finite measurable partitions ξ and η set

$$d_R(\xi, \eta) = H_\mu(\xi|\eta) + H_\mu(\eta|\xi).$$

Show that d_R is a metric on the set of (all equivalence classes $\mod 0$ of) finite measurable partitions. *It is called Rokhlin metric.*

Problem 3.

Let (M, μ) be a measure space with probability measure μ. For two finite measurable partitions ξ and η by adding null sets if necessary we may assume that ξ and η have the same number of elements. Consider the set of bijections σ between elements of ξ and η and set

$$d_\Delta(\xi, \eta) = \min_\sigma \sum_{C \in \xi} \mu(C \Delta \sigma(C)).$$

Show that d_Δ is a metric on the set of (all equivalence classes $\mod 0$ of) finite measurable partitions.

Problem 4.

Prove that convergence in d_Δ implies convergence in d_R.

Problem 5.

Define $T : [0, 1] \to [0, 1]$ by

$$T(x) = \begin{cases} 2x, & \text{if } x \in [0, 1/2]; \\ 2 - 2x, & \text{if } x \in (1/2, 1]. \end{cases}$$

Find $h_{top}(T)$.