ELEMENTARY ANALYSIS 140A

Final Exam SAMPLE

Problem 1.

Prove that for every $n \geq 1$, $2^{2^n} - 1$ is divisible by at least n distinct primes.

Problem 2.

Determine the limit of the sequence $\left\{5 - \frac{1}{n} + \frac{1}{n^2}\right\}_{n\in\mathbb{N}}$. Prove that the sequence converges to that limit using the definition of sequence convergence.

Problem 3.

If possible, give an example of each of the following. Write "not possible" when appropriate.

a) A sequence $\{s_n\}$ with $\limsup s_n = +\infty$ and $\liminf s_n = 0$.

b) A bounded sequence which diverges.

c) A series $\sum a_n$ which diverges, but for which the series $\sum a_n^2$ converges.

d) A continuous but not uniformly continuous function $f : [-2000, 2000] \to \mathbb{R}$.

e) A function $f : \mathbb{R} \to \mathbb{R}$ which is continuous at exactly one point (and discontinuous at every other point).

Problem 4.

Prove that if the series $\sum a_n$ converges absolutely then the series $\sum(-1)^n a_n^2$ also converges.

Problem 5.

Give an example of a metric space (X, d) for which there exists a continuous unbounded function $f : X \to \mathbb{R}$. Is it possible to take the standard Cantor set as such an example?

Problem 6.

Prove that $e^{-x} = x$ for some $x > 0$.

Problem 7.

Prove or Disprove: $f(x) = x^2 \sin \frac{1}{x^2}$ is uniformly continuous on $(0, 5)$.