Problem 1.

Find the limits of the following sequences (no proof is required):

\[ \lim_{n \to \infty} \frac{n^3 + n^2 + n + 1}{3n^3 + 2n^2 + n + 1} = \]

\[ \lim_{n \to \infty} (2n)^{1/n} = \]

\[ \lim_{n \to \infty} \left( n^{1/n} - \frac{1}{2} \right)^n = \]

Problem 2.

For each sequence, find the set of subsequential limits:

1) \( s_n = (-1)^n + \frac{1}{2^n} \)

2) \( s_n = \cos \left( \frac{\pi}{4} + \frac{n\pi}{2} \right) \)

3) \( s_n = (-1)^n + \frac{1}{2^n} + \cos \left( \frac{\pi}{4} + \frac{n\pi}{2} \right) \)

Problem 3.

Prove that the closure of an interior of any closed set \( E \subset \mathbb{R} \) is a subset of \( E \). Does it have to coincide with \( E \)?
Problem 4.

For each of the following sets find its closure:

1) \( E = \{ \frac{1}{n} + \frac{1}{2^n} \mid n \in \mathbb{N} \} \)

2) \( E = \mathbb{Q} \cap (0, 1) \)

3) \( E = \mathbb{Q} \cap \{ x \in \mathbb{R} \mid x^2 < 2 \} \)

Problem 5.

Consider a metric space \((X, d)\), where \(X = \{x, y, z, u\}\) and
\[
\begin{align*}
    d(x, y) &= d(x, z) = d(x, u) = d(y, x) = d(z, x) = d(u, x) = 2, \\
    d(y, z) &= d(y, u) = d(z, u) = d(z, y) = d(u, y) = d(u, z) = 1, \\
    d(x, x) &= d(y, y) = d(z, z) = d(u, u) = 0.
\end{align*}
\]

Check that \(d\) is indeed a metric, and prove that \((X, d)\) is a complete metric space.