Final Exam

Wednesday, December 10, 2008 — 8:00 am - 10:00 am

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Student’s name:
Problem 1.

Find a bounded solution of the differential equation

\[ \frac{dy}{dt} = y - \sin t. \]
Problem 2.

Compute the first three Picard iterates \((y_0(t), y_1(t), \text{ and } y_2(t))\) for the initial value problem

\[
\frac{dy}{dt} = y + e^{y-1}, \quad y(0) = 1.
\]
Problem 3.

Solve the initial value problem

\[ y'' + 3y' + 2y = -1 - H_1(t), \quad y(0) = 0, \quad y'(0) = 0, \]

where

\[ H_1(t) = \begin{cases} 
0, & t < 1, \\
1, & t \geq 1. 
\end{cases} \]
Problem 4.

Check that $y_1(t) = t$ and $y_2(t) = e^t + t - 1$ are solutions of the equation

$$(e^t + 1)y'' - 2y' - e^ty = -te^t - 2.$$ 

Find the general solution.
Problem 5.

Find the general solution of the system of equations

\[
\begin{align*}
\frac{dx_1}{dt} &= x_1 - x_2 + x_3 \\
\frac{dx_2}{dt} &= x_1 + x_2 - x_3 \\
\frac{dx_3}{dt} &= 2x_1 - x_2
\end{align*}
\]