Introduction to differential equations.

Midterm Review.

- Differential equations of the first order
  - Linear
  - Separable
  - Exact
  - Homogeneous
  - The existence and uniqueness theorem

- Differential equations of the second order
  - Linear homogeneous and non-homogeneous, properties of solutions, Wronskian
  - Linear homogeneous with constant coefficients
  - Linear non-homogeneous, method of variation of parameters
  - Linear non-homogeneous, with constant coefficients and special r.h.s. (polynomial, exponential, function, $\sin x$, $\cos x$, or product of these)
  - Reduction of order

Midterm: 4 problems
2 - first order
2 - second order
One of the problems is "theoretical."
40 points + 1 (evaluation form)
Problem 1

\[ y' + a(t)y = f(t), \]

\[ a(t), f(t) \text{ are continuous for } t \in \mathbb{R}, \]

\[ a(t) \geq c > 0 \]

\[ f(t) \to 0 \text{ as } t \to \infty. \]

Prove that for every solution \( y(t) \)

\[ y(t) \to 0 \text{ as } t \to \infty. \]

Problem 2

Find a continuous solution of the IVP

\[ y' + y = g(t), \quad y(0) = 0 \]

\[ g(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ 1, & t > 1 \end{cases} \]

Problem 3

Is it possible that the graphs of two different solutions of the equation

\[ y' = t + y^2 \]

intersect at some point \((t_0, y_0)\)?

The same question for

\[ y'' = t + y^2. \]
Problem 4

\[ y'' + p(t)y' + q(t)y = 0 \]

Assume that \( q(t) < 0 \). Prove that a solution \( y(t) \) cannot have a positive maximum.

Problem 5

A linear non-homogeneous equation of second order has solutions \( y_1 = 1, y_2 = t, y_3 = t^2 \).
Find a general solution.

Problem 6

For which \( a, b \)

every solution of the equation

\[ y'' + ay' + by = 0 \]

has infinite number of zeros?

Problem 7

For which \( k, \omega \neq 0 \)

the equation \( y'' + k^2 y = \pm \omega^2 \)

has at least one periodic solution?
Problem 8

Find a general solution

\[ y'' = 2 + y' \]

Problem 9

Is it possible that the graphs of two solutions of the differential equation

\[ y'' + q(t)y = 0, \quad y(t) \text{ and } y''(t) \text{ continuous,} \]

have the form:

a)

\[ y \]

\[ t \]

b)

\[ y \]

\[ t \]