9:00 AM

① $x^2y^3 + y + (x^3y^2 + x)y' = 0$

check for exactness:

$$\left.\begin{array}{l}\frac{\partial}{\partial y}(x^2y^3 + y) = 3x^2y^2 + 1 \\ \frac{\partial}{\partial x}(x^3y^2 + x) = 3x^2y^2 + 1\end{array}\right] =$$

$\Rightarrow$ exact.

$\Rightarrow \exists\, \phi \in C^2(\mathbb{R}^2)$ s.t. $\frac{\partial \phi}{\partial x} = x^2y^3 + y$ (i)

$\frac{\partial \phi}{\partial y} = x^3y^2 + x$ (ii)

(i) $\Leftrightarrow$ $\phi = \int (x^2y^3 + y)\,dx + h(y)$, some $h \in C^2(\mathbb{R}^2)$

$= \frac{1}{3}x^3y^3 + yx + h(y)$

$\overset{(ii)}{\Rightarrow}$ $\cancel{x^3y^2 + x} = \frac{\partial \phi}{\partial y} = \cancel{x^3y^2 + x} + h'(y)$

$\Rightarrow h'(y) = 0 \Rightarrow h(y) = \tilde{C}$, some $\tilde{C} \in \mathbb{R}$

Thus, $\phi = \frac{1}{3}x^3y^3 + yx + \tilde{C}$, $\tilde{C} \in \mathbb{R}$ and

therefore:

general sol. of (1):

$$\underline{\underline{\frac{1}{3}x^3y^3 + yx = C \quad , \; C \in \mathbb{R}.}}$$