Exercise 28.10, page 212

Let $h(x) = [\cos x + e^x]^{12}$

(a) Calculate $h'(x)$.

Solution: $h'(x) = 12[\cos x + e^x]^{11}(-\sin x + e^x)$

(b) Show how the chain rule justifies your computation in part (a) by writing $h = g \circ f$ for suitable $f$ and $g$.

Solution: $g(x) = x^{12}, f(x) = \cos x + e^x$.

Exercise 28.12, page 213

(a) Differentiate the function whose value at $x$ is $\cos(e^{x^5-3x})$.

Solution: $-\sin(e^{x^5-3x})(e^{x^5-3x})(5x^4 - 3)$.

(b) Use Exercise 28.11 or Theorem 28.4 to justify your computation in part (a).

Solution: Let $h(x) = \cos x, g(x) = e^x, f(x) = x^5 - 3x$. By Exercise 28.11,

$$(h \circ g \circ f)'(x) = h'(g \circ f(x))g'(f(x))f'(x).$$

Alternatively, by Theorem 28.4, writing $h \circ g \circ f = h \circ (g \circ f)$ we have $(g \circ f)'(x) = g'(f(x))f'(x)$ and

$$(h \circ g \circ f)'(x) = [h \circ (g \circ f)]'(x) = h'(g \circ f(x))(g \circ f)'(x) = h'(g \circ f(x))g'(f(x))f'(x).$$

Exercise 28.14(b), page 213

Suppose that $f$ is differentiable at $a$. Prove that

$$\lim_{h \to 0} \frac{f(a + h) - f(a - h)}{2h} = f'(a).$$

Solution:

$$\frac{f(a + h) - f(a - h)}{2h} = \frac{f(a + h) - f(a)}{2h} + \frac{f(a - h) - f(a)}{-2h} \to f'(a)/2 + f'(a)/2.$$